Problem 16)

a) Using $f(x)=\int_{-\infty}^{\infty}F(s)\exp(i2\pi sx)ds$ and $G(s)=\int_{-\infty}^{\infty}g(x)\exp(-i2\pi sx)dx$, we write

$$\int_{-\infty}^{\infty}f(x)g^*(x)dx=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}F(s)\exp(i2\pi sx)ds\right)g^*(x)dx$$

Changing order of integration →
$$=\int_{-\infty}^{\infty}F(s)\left(\int_{-\infty}^{\infty}g^*(x)\exp(i2\pi sx)dx\right)ds=\int_{-\infty}^{\infty}F(s)\left(\int_{-\infty}^{\infty}g(x)\exp(-i2\pi sx)dx\right)^* ds$$

$$=\int_{-\infty}^{\infty}F(s)G^*(s)ds.$$

b) $\int_{-\infty}^{\infty}\mathrm{sinc}^3(s)\,ds=\int_{-\infty}^{\infty}\mathrm{sinc}(s)\mathrm{sinc}^2(s)\,ds=\int_{-\infty}^{\infty}\mathrm{Rect}(x)\mathrm{Tri}(x)\,dx=2\int_0^{1/2}(1-x)\,dx=\frac{3}{4}.$

c) $\int_{-\infty}^{\infty}\mathrm{sinc}^4(s)\,ds=\int_{-\infty}^{\infty}\mathrm{sinc}^2(s)\mathrm{sinc}^2(s)\,ds=\int_{-\infty}^{\infty}\mathrm{Tri}(x)\mathrm{Tri}(x)\,dx=2\int_0^1(1-x)^2dx=2\int_0^1 y^2dy=\frac{2}{3}.$

d) $\int_0^{\infty}\exp(-x)\,\mathrm{sinc}(x)dx=\frac{1}{2}\int_{-\infty}^{\infty}\exp(-|x|)\,\mathrm{sinc}(x)dx=\int_{-\infty}^{\infty}\frac{\mathrm{Rect}(s)}{1+(2\pi s)^2}ds=2\int_0^{1/2}\frac{ds}{1+(2\pi s)^2}$

Change of variable $2\pi s=\tan\theta$ →
$$=\frac{2}{2\pi}\int_0^{\tan^{-1}\pi}\frac{(1+\tan^2\theta)d\theta}{1+\tan^2\theta}=\frac{1}{\pi}\int_0^{\tan^{-1}\pi}d\theta=\frac{\tan^{-1}\pi}{\pi}.$$